Quiz 2
Business Mathematics

7^{th} February 2006

Time: 1 hour (10–11pm)

1. Evaluate the following integrals. [10]

$$\int_0^1 \pi\,dx, \qquad \int_0^2 \left(x^2-3\right)\,dx, \qquad \int_{-1}^1 \frac{1}{z}\,dz, \qquad \int_0^2 \left(x^2+\sqrt{x}\right)\,dx, \qquad \int_0^1 xe^x\,dx$$

Solution.

$$\int_0^1 dx = \pi(x|_0^1 = \pi$$

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$$\int_0^2 \left(x^2-3\right)\,dx = \left(\frac{x^3}{3}-3x\right|_0^2 = \frac{8}{3}-6 = -\frac{10}{3}$$

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In $\int_{-1}^1 \frac{1}{z}dz$ the limits of integration pass through 0, where there is discontinuity and where $1/z$ is undefined. Further, $\lim_{z\to 0^-} f(z) = -\infty$ and $\lim_{z\to 0^+} f(z) = \infty$. We look at

$$\int_0^1 \frac{1}{z}\,dz = \lim_{a\to 0^+} (\ln z|_a^1 = \lim_{a\to 0^+} (\ln 1 - \ln a) = 0 - (-\infty) = \infty$$

and

$$\int_{-1}^1 \frac{1}{z}dz = \int_{-1}^0 \frac{1}{z}dz + \int_0^1 \frac{1}{z}dz$$

where

$$\int_{-1}^0 \frac{1}{z}dz = \lim_{b\to 0^-} \int_{-1}^b \frac{1}{(-z)}d(-z) = \lim_{b\to 0^-} (\ln(-z)|_{-1}^b = \lim_{b\to 0^-} (\ln b - \ln(1)) = \lim_{b\to 0^-} (\ln b)$$

the latter of which is undefined. Therefore $\int_{-1}^1 \frac{1}{z}dz$ is also undefined.

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$$\int_0^2 \left(x^2+\sqrt{x}\right)\,dx = \left(\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right|_0^2 = \frac{8}{3}+\frac{2\sqrt{8}}{3} = \frac{8+4\sqrt{2}}{3}$$

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Let $u = x$ and $dv = e^x\,dx$. Then $du = dx$ and $v = e^x$. And then,

$$\int_0^1 xe^x\,dx = (xe^x|_0^1 - \int_0^1 e^x\,dx = (xe^x - e^x|_0^1 = 1$$

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